

ALFVÉN WAVE-DRIVEN SUPERNOVA EXPLOSION

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ABSTRACT

We investigate the role of Alfvén waves in the core-collapse supernova (SN) explosion. We assume that Alfvén waves are generated by convections inside a proto-neutron star (PNS) and emitted from its surface. Then these waves propagate outwards, dissipate via nonlinear processes, and heat up matter around a stalled prompt shock. To quantitatively assess the importance of this process for the revival of the stalled shock, we perform 1D time-dependent hydrodynamical simulations, taking into account the heating via the dissipation of Alfvén waves that propagate radially outwards along open flux tubes. We show that the shock revival occurs if the surface field strength is larger than $\sim 2 \times 10^{15}$ G and if the amplitude of velocity fluctuation at the PNS surface is larger than $\sim 20\%$ of the local sound speed. Interestingly, the Alfvén wave mechanism is self-regulating in the sense that the explosion energy is not very sensitive to the surface field strength and initial amplitude of Alfvén waves as long as they are larger than the threshold values given above.

Subject headings: MHD – supernovae:general – waves

1. INTRODUCTION

The most promising scenario of collapse-driven supernova (SN) is currently supposed to be the so-called delayed explosion by neutrino heating (Kotake et al. 2006; Janka et al. 2007, and references therein): the prompt shock, which was generated by core bounce and stalled by neutrino emissions and dissociations of nuclei, is heated up and revived by neutrinos coming out of the proto-neutron star (PNS), leading eventually to a SN explosion. Under spherical symmetry, however, no successful explosion has been obtained so far even though up-to-date micro-physics, such as equation of state and weak interaction rates, have been fully incorporated (Janka et al. 2007). Various other effects have also been explored over the years. The implications of stellar rotation and different sorts of hydrodynamical instabilities have been extensively studied. (Kotake et al. (2006) and see also Marek & Janka (2007); Mezzacappa et al. (2007) for very recent progresses.)

Magnetic fields are drawing much attention of researchers these days although the history of research is quite long. After some pioneering papers (LeBlanc & Wilson 1970; Bisnovatyi-Kogan, Popov. & Samokhin 1976; Meier et al. 1976; Symbalisty 1984), the subject had been forgotten for a while because it was realized that very strong magnetic fields are required to affect the supernova dynamics, which was supposed to be unrealistic at that time. The situation changed with the observational evidence of strongly magnetized neutron stars or magnetars (Thompson & Duncan 1996) and the

progress in theoretical understanding of the magneto-rotational instability or MRI (Balbus & Hawley 1991).

The possible importance of MRI in the generation of magnetic field in core-collapse SNe was first pointed out by Akiyama et al. (2003). A lot of papers have been published (Yamada & Sawai 2004; Thompson, Quataert, & Burrows 2005; Moiseenko, Bisnovatyi-Kogan, & Ardeljan 2006; Kotake et al. 2006; Burrows et al. 2007; Wheeler & Akiyama 2007) to investigate possible roles of magnetic fields in the supernova dynamics, especially on their kinematical aspects such as magnetic pressure and torque induced by rapid rotations in supernova cores. Thompson, Quataert, & Burrows (2005), on the other hand, considered a MHD turbulence that is possibly induced by MRI as a source of viscosity to tap free energies stored in differential rotations. Although the rapid rotation of stellar core is prerequisite in these studies, it may not be so easy to obtain in the presence of magnetic fields according to recent stellar evolution models because the transfer of angular momentum is efficient and the core may rotate rather slowly just prior to the gravitational collapse (Heger, Woosley, & Spruit 2005)⁵.

More recently, yet another supernova mechanism was put forward: sound waves generated by PNS oscillations of mainly g-mode nature, which are probably induced by turbulence caused by the standing accretion shock instability (SASI), heat up matter through nonlinear dissipations, revive the stalled shock wave and produce explosions at very late times (Burrows et al. 2006). Since this acoustic mechanism consists of several steps, whose efficiencies appear to be not very high (Yoshida, Ohnishi, & Yamada 2007; Marek & Janka 2007), the viability of the mechanism is still controversial and further explorations from various

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⁵ It is worth noting that Blondin & Mezzacappa (2007) recently pointed out a new mechanism to generate the rotation of PNSs. They claimed that the non-axisymmetric SASI might be a source of the angular momentum of PNSs. If true, no rotation may be needed prior to the collapse to account for the spin of neutron stars.

view points are needed.

At present the supernova mechanism is still elusive in spite of these extensive efforts. The above-mentioned research trend naturally leads us to the exploration of still another type of waves that may also contribute to the supernova explosion: Alfvén waves. If the supernova core is magnetized, the oscillations of PNS will emit not only sound waves but also Alfvén waves. It is also possible that Alfvén waves are excited by convections in PNS, which have been demonstrated to exist after core bounce (Keil, Janka, & Müller 1996). This is quite analogous to what happens in the Sun (e.g. Suzuki & Inutsuka 2005, 2006, hereafter SI05; SI06). A fraction of these waves propagate outwards and dissipate later through nonlinear processes, which then will heat up matter and may lead to the revival of the stalled shock. Such a scenario as a supernova mechanism has not been studied on a quantitative basis so far, although it has been discussed in a qualitative manner (e.g. Wheeler et al. 2000; Woosley & Janka 2005; Burrows et al. 2006) or in the context of the nucleosynthesis in ν -driven winds (Suzuki & Nagataki 2005). It should be emphasized that it is the dissipations of fluctuating components of magnetic fields associated with Alfvén waves that heat up matter and revive the shock wave, whereas in most of the previous papers, torques and pressures exerted by total magnetic fields were the key players. Since the main aim in this paper is not to construct a realistic model but to elucidate the characteristics of the Alfvén wave heating in a quantitative manner, we employ 1D simplified but dynamical simulations.

2. MODELS

In this section, we describe the scenario we have in mind and explain nonlinear dissipation processes of Alfvén waves. Based on these we give the basic equations and employed approximations in detail.

2.1. Basic Picture

Convections around the neutrinosphere that roughly coincides with the surface of a PNS are induced by negative gradients of lepton-fraction and entropy about a few tens of milliseconds after core bounce. According to hydrodynamical simulations of PNS convections by Keil, Janka, & Müller (1996), the amplitude of velocity fluctuations near the PNS surface is $\simeq 4 \times 10^3 \text{ km s}^{-1}$ on average and becomes as high as 10^4 km s^{-1} . Recent simulations also reported that SASI in accreting flows might excite a comparable order of surface fluctuations (Ohnishi, Kotake, & Yamada 2006). And not to mention, the non-radial oscillations of PNS are an obvious possibility as a source of Alfvén waves.

These activities in PNS generate various modes of waves with the amplitude of velocity fluctuations mentioned above, which then emanate from its surface and propagate outwards. If the magnetic field is sufficiently strong, two types of magnetic waves are important: Alfvén wave and fast wave. The Alfvén waves propagate along field lines and are less subject to dissipations thanks to their incompressive character. On the other hand, the fast waves propagate almost isotropically and can traverse field lines although they suffer more damping owing to the compressive nature. There-

fore, Alfvén waves can transport energy farther away along magnetic field lines unless they form closed loops. The fast waves could become important, however, in the equatorial region if PNS rotates rapidly and the field lines are tightly wound up. In this paper, we assume that open-field regions prevail in the post-bounce core SN core and study the energy deposition by the dissipations of Alfvén waves through nonlinear processes and its implications for the shock revival.

The energy source in this mechanism is eventually the gravitational energy that is released by the matter accretion onto PNS. Some of the released gravitational energy is converted to the kinetic energy of surface fluctuations in the PNS by convections, SASI and other instabilities. Then the surface fluctuations excite various waves in PNS, some of which are Alfvén waves traveling outwards along field lines. This sequence of processes is quite similar to what Burrows et al. (2006) proposed in their acoustic mechanism.

As will be shown later, we assume strong magnetic fields of the magnetar scale, $\sim 10^{15} \text{ G}$, on the PNS surface. We do not specify the origin of the strong magnetic fields in this paper (but see §4.1 for related discussions.) and treat the field strength as a free parameter. As mentioned above, we focus on the propagation of Alfvén waves and pay attention only to magnetic flux tubes that are extended beyond the stalled shock because the Alfvén wave carry the energy parallel to a field line; if the magnetic field is closed near the PNS surface, Alfvén waves cannot transfer energy to the stalled shock.

Since, as a first step, the aim of the paper is not to construct a realistic model, which should be the next step, but to understand the essential features of the Alfvén wave heating in the post bounce core, we make the models as simple as possible, incorporating only minimum ingredients. Since stellar rotation is not an indispensable player in the mechanism considered here, we just neglect it and assume radial magnetic fields as the simplest configuration for open field lines in this paper. The field strength is given by

$$B_r = B_{r,0} \frac{r_0^2}{r^2}, \quad (1)$$

where $B_{r,0}$ is the field strength at the PNS surface, $r = r_0$. We perform one-dimensional (1D) simulations, ignoring the effects of neighboring magnetic fields.

Even if the stellar rotation is fast, our models will be still applicable locally to the polar region, where the centrifugal force is minimum, although the simple prescription for the expansion factor of flux tubes, $\propto r^2$, adopted in this paper may need further elaboration. In the equatorial region, on the other hand, the validity of the assumption in this paper depends not only on the rotation period but also on the reconnection efficiency among the field lines frozen into the accreted matter. This issue will be discussed again in §4.1.

2.2. Dissipation of Alfvén Waves

The effects of the Alfvén waves on accreting matter are twofold: (1) the heating of matter by the Alfvén wave dissipations and (2) the extra pressure exerted by the Alfvén waves. We take into account only the former in this paper. The Alfvén wave with linear amplitude is

non-dissipative due to the incompressive nature. However, nonlinear Alfvén waves suffer various dissipation processes. The nonlinearity, w , of Alfvén waves can be defined as

$$w \equiv \frac{\delta v_{\perp}}{v_A} = \frac{\delta B_{\perp}}{B_r}, \quad (2)$$

where δv_{\perp} and δB_{\perp} are the amplitudes of velocity and magnetic field, the subscript ' \perp ' denotes the transverse directions with respect to B_r , and $v_A = B_r/\sqrt{4\pi\rho}$ is the Alfvén speed. Here we have used the relation, $\delta v_{\perp} = \delta B_{\perp}/\sqrt{4\pi\rho}$, that the Alfvén wave satisfies (§10 of Lamers & Cassinelli 1999). Even though the initial amplitude at the launch from the PNS surface is in the linear regime, $w \ll 1$, it grows thanks to the decrease of B_r (Equation (1)). Eventually, the Alfvén wave becomes nonlinear, $w \sim 1$, and various dissipation processes set in.

The excitation of compressive waves by nonlinear mode conversions is a route of the dissipation. If the Alfvén wave is not strictly circularly polarized, the magnetic pressure, $\delta B_{\perp}^2/8\pi$, fluctuates along with B_r and induces longitudinal compressive motions, most of which correspond to slow magnetohydrodynamical (MHD) waves (SI05;SI06). Even if the Alfvén wave is circularly polarized, it is subject to the parametric decay instability, which generates outgoing slow MHD waves and incoming Alfvén waves (Goldstein 1978; Terasawa et al. 1986). The velocity amplitudes of the slow waves are also amplified as they propagate outwards and the density decreases. Eventually the wave fronts steepen to form shocks and heat up matter.

If B_r has a transverse gradient (along \perp direction), fast MHD waves that propagate perpendicularly to B_r are also excited from Alfvén waves (Nakariakov, Roberts, & Murawski 1997). These fast waves also heat ambient matter by the shock formation. Moreover, the transverse inhomogeneity of B_r leads to the phase mixing (Heyvaerts & Priest 1983), which is another channel of the dissipation of Alfvén waves.

The turbulent cascade may also work in the dissipation of Alfvén waves. In the PNS, the Alfvén speed is not constant along the radial magnetic field because of the changes in both B_r and density. Then, incoming Alfvén waves are excited from the outgoing ones by the deformation of wave shapes and the interactions between the outgoing and incoming Alfvén waves lead to the formation of smaller scale (large wave number) structures mainly in the perpendicular directions (Goldreich & Sridhar 1995). This turbulent cascade to higher wave number proceeds up to the dissipation range, where resistivities (for magnetic field fluctuations) and/or viscosities (for velocity fluctuations) become important. Then, the energy that Alfvén waves are carrying is finally transferred to the ambient matter.

As a result of these various dissipation processes, the nonlinearity, w , of the outgoing Alfvén waves is saturated at a certain level. According to dynamical simulations of solar (SI05;SI06) and stellar (Suzuki 2007) winds, the saturation level is found to be $w \lesssim 0.3-1$, which we apply in this paper to the Alfvén waves in the supernova core. It should be noted that the saturation with a constant w implies that δB_{\perp} itself decreases as the Alfvén wave propagate further outwards, since B_r is declining (see

Equation (2)). This means the dissipation of the energy of Alfvén waves and results in the heating of the matter.

2.3. Formulation

We evaluate the Alfvén wave heating by considering the conservation of wave energy under the WKB, or short wave length, approximation. This treatment enables us to obtain the estimations in a simple manner without solving the fully nonlinear MHD equations. The conservation of wave energy can be expressed (Jacques 1977) as

$$\frac{\partial \mathcal{E}_w}{\partial t} + \nabla \cdot \mathbf{F}_w - \mathbf{v} \cdot \nabla P_w = -\rho \dot{q}_w, \quad (3)$$

where ρ , \mathbf{v} are the density and velocity of the accretion flow and \mathcal{E}_w is the wave energy density, \mathbf{F}_w is the wave energy flux, P_w is the wave pressure, and \dot{q}_w is the dissipation rate of wave energy per unit mass. As for the Alfvén wave, $\mathcal{E}_w = \rho \langle \delta v_{\perp}^2 \rangle = \langle \delta B_{\perp}^2 \rangle / 4\pi$ and $P_w = \mathcal{E}_w / 2$, where the bracket $\langle \dots \rangle$ stands for the average over a period of the Alfvén wave. The last term on the left hand side denotes the rate of work done by the Alfvén waves on the accretion flow, which we neglect in this paper as stated above. The right hand side represents the energy deposition by the Alfvén waves to the accreting matter. When $\dot{q}_w > 0$, the matter is heated up by the wave dissipations, whereas the Alfvén waves travel without dissipation if $\dot{q}_w = 0$.

It is convenient for later use to introduce an adiabatic constant, the so-called wave action, \mathbf{H}_w , defined as follows (Jacques 1977):

$$\nabla \cdot \mathbf{F}_w - v_r \frac{dP_w}{dr} \equiv \frac{v_A}{v_A + v} \nabla \cdot \mathbf{H}_w. \quad (4)$$

We neglect relativistic corrections because they are minor outside the PNS. It should be noted that the wave action, \mathbf{H}_w , instead of \mathbf{F}_w , is conserved in moving media. The specific form of \mathbf{H}_w is given as (Jacques 1977)

$$\mathbf{H}_w = \frac{\langle \delta B_{\perp}^2 \rangle}{4\pi} \frac{(v_A + v)(\mathbf{v}_A + \mathbf{v})}{v_A}. \quad (5)$$

In this paper we assume the steady propagations of Alfvén waves and neglect the time derivative, $\frac{\partial}{\partial t}$, in Equation (3). This is valid when the Alfvén transit time is shorter than a typical time-scale of the system, which will be discussed in §4.2.

The initial amplitude of Alfvén waves at the PNS surface is supposed to be an order of convective velocities at the surface of PNS, which are suggested to be a fraction of the sound speed⁶:

$$\delta v_{\perp,0} = \epsilon c_{s,0}, \quad (6)$$

where $c_{s,0}$ is the sound speed at the PNS surface and we choose $\epsilon = 0.1 - 0.3$ in our simulations. Because $c_{s,0}$ is $\approx 10\%$ of the light speed, $\epsilon = 0.1 - 0.3$ corresponds to $\delta v_{\perp,0} = (0.3 - 1) \times 10^4 \text{ km s}^{-1}$, which is comparable to the values obtained in hydrodynamical simulations (Keil, Janka, & Müller 1996). For a sufficiently large background magnetic field at the PNS surface, $B_{r,0} \gtrsim 5 \times 10^{14} \text{ G}$, the initial wave amplitude is

⁶ This is very similar to the surface convection in the Sun. The observed granulation speed is $1 - 2 \text{ km s}^{-1}$, while the sound speed at the photosphere is $\approx 5 \text{ km s}^{-1}$.

small in the sense that $w < 0.1$. In such a condition, the Alfvén wave travels outwards without dissipation ($\dot{q}_w = 0$) near the PNS surface and its amplitude evolves according to the following relation:

$$\frac{(\delta B_\perp^2)}{4\pi} \frac{(v_A + v_r)^2}{v_A} r^2 = H_{w,0} r_0^2, \quad (7)$$

where $H_{w,0}$ is the wave action at its surface, and v_r is the radial velocity of the background accretion flow.

As the Alfvén waves travel outwards, the nonlinearity, w , increases because of the expansion of the radial magnetic flux tube (Equation (1)). The dissipation of Alfvén waves eventually sets in when they reach the nonlinear regime as discussed above and the nonlinearity of the Alfvén waves is saturated at a certain level:

$$w \approx \alpha. \quad (8)$$

In this paper we adopt a constant $\alpha = 0.5$ as a standard saturation level, based on our previous results on the solar and stellar winds (SI05;SI06;Suzuki 2007), which showed that $w (\lesssim 1)$ is more or less constant or very slowly varying as a function of r (see also e.g. Hollweg 1973, for steady-state modeling). This simple prescription of the constant saturation level is expected to incorporate phenomenologically all the complex physical processes of Alfvén wave dissipations discussed in §2.2 and provide us with a reasonable heating rate.

For the steady state, the energy dissipation rate, \dot{q}_w , can be obtained from the conservation of the wave action without referring to the details of the nonlinear processes that are responsible for the dissipations. Using the relation in the dissipation region, $\delta B_\perp \propto B_r \propto (r_0/r)^2$ (Equation (1)), the heating rate, \dot{q}_w (erg $\text{g}^{-1}\text{s}^{-1}$), can be expressed as

$$\dot{q}_w = \frac{v_A}{v_A + v_r} \frac{\alpha^2 B_r}{4\pi\rho} \frac{d}{dr} ((v_A + v_r)^2 \sqrt{4\pi\rho}). \quad (9)$$

An important point here is that we can evaluate \dot{q}_w from the local distributions of ρ , v_r , and B_r .

2.4. Simulations

Incorporating the above formula for the heating by Alfvén waves, we perform 1D time-dependent simulations of the post-bounce evolutions of SN core. We use the $15M_\odot$ progenitor star of Woosley & Weaver (1995) as an initial condition, and employ a numerical code (Sumiyoshi et al. 2005) to solve general relativistic hydrodynamics and neutrino transport, adding the extra heating term given by Equation (9) in the energy equation. The Alfvén wave heating (\dot{q}_w) is switched on at 100 ms after core bounce. We have in mind that this is the time for the development of (magneto-)convection that drives Alfvén waves (Keil, Janka, & Müller 1996; Akiyama et al. 2003; Masada, Sano, & Takabe 2006). Incidentally, we define the PNS surface as the position of the density, $\rho_0 = 10^{11} \text{ g cm}^{-3}$, which, as a consequence, becomes smaller as the PNS contracts.

We have three parameters, $B_{r,0}$, ϵ , and α in the above prescription. We fix the saturation level, $\alpha = 0.5$, which controls the location of wave dissipations. The other two, the field strength, $B_{r,0}$, at the PNS surface and the normalized initial amplitude, ϵ , of perturbations, determine the wave energy injected from the PNS surface. In the following, we explore the condition on these two parameters for the shock revival by simulating nine models in Table 1.

TABLE 1
SUMMARY OF SIMULATIONS.

Model	$B_{r,0}(\text{G})$	ϵ	Explosion	E_{exp}	M_{ej}	M_{cut}
I	1×10^{15}	0.1	No	—	—	—
II	1×10^{15}	0.2	No	—	—	—
III	1×10^{15}	0.3	No	—	—	—
IV	2×10^{15}	0.1	Marginal	—	—	—
V	2×10^{15}	0.2	Yes	1.2	0.08	1.38
VI	2×10^{15}	0.3	Yes	1.6	0.10	1.37
VII	3×10^{15}	0.1	Yes	0.33	0.04	1.41
VIII	3×10^{15}	0.2	Yes	1.5	0.10	1.38
IX	3×10^{15}	0.3	Yes	2.2	0.14	1.38

NOTE. — The explosion energy, ejecta mass and PNS mass are denoted by E_{exp} , M_{ej} and M_{cut} , respectively. The unit of E_{exp} is 10^{51} erg and the units of M_{ej} and M_{cut} are M_\odot .

3. RESULTS

Figure 1 displays the success or failure of shock revival in the plane of the field strength at the PNS surface, $B_{r,0}$, and the initial amplitude of velocity perturbation, ϵ . Table 1 gives more detailed information on the ejected mass, M_{ej} , explosion energy, E_{exp} , and remnant (\approx PNS) mass, M_{cut} , for the models that obtain the shock revival under the current approximation. They are estimated at the time $t = 200$ ms after core bounce as follows. We first define the ejecta as the collection of mass shells with positive total energy, which is the sum of kinetic, internal and gravitational energies. Then M_{ej} and E_{exp} are obtained as the sums of mass and total energy, respectively, of each mass shell that comprises the ejecta. In so doing, the non-relativistic expression is employed for the energy. It is clear that an explosion with $E_{\text{exp}} \geq 10^{51}$ erg is obtained if the magnetic field at the PNS surface is strong, $B_{r,0} \gtrsim 2 \times 10^{15} \text{ G}$, and if the initial amplitude of Alfvén waves is larger than a certain value, $\epsilon (= \delta v_\perp / c_s) \gtrsim 0.2$. Note that the surface field strength is of the same order as those inferred for magnetars.

Figure 2 presents a typical Alfvén wave-driven shock revival, in which the result for model V (red lines) is superimposed on the original result for the non-magnetized spherically symmetric model (black lines) (Sumiyoshi et al. 2005). The trajectories of mass shells are plotted against the time from core bounce. The figure clearly demonstrates the shock revival by the Alfvén wave heating for the otherwise failed neutrino heating model (Sumiyoshi et al. 2005) like those discussed in many previous papers (e.g. Kotake et al. 2006, and references therein).

Figure 3 shows the rates of Alfvén wave heating and neutrino heating (top panel) along with the velocity distribution (bottom panel) at $t = 100$ ms. The top panel demonstrates that the Alfvén wave heating operates mainly in the vicinity of the stalled shock wave and dominates the neutrino heating. The main reason for the localization of the Alfvén wave heating in the *Eulerian* frame is the trapping of Alfvén waves. The propagation speed of the outgoing Alfvén wave is $v_A + v_r$ in this frame. It rapidly decreases from $2 \times 10^4 \text{ km s}^{-1}$ at $r = 100 \text{ km}$ to $\approx 0 \text{ km s}^{-1}$ at $r = 300 \text{ km}$ for model V (bottom panel of Figure 3), for example. The Alfvén waves cannot travel further outwards and are trapped inside $r \lesssim 300 \text{ km}$ in model V, so that they spend a long time there to damp almost completely. In the *Lagrangian* frame that moves at

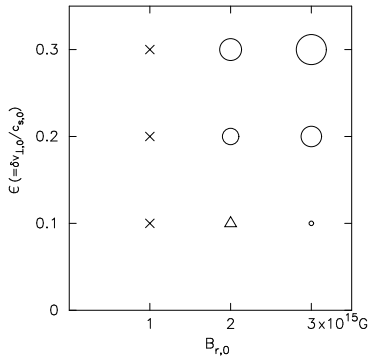


FIG. 1.— Status of each model. The circles and crosses respectively corresponds to the models that result in shock revival and the models that do not give shock revival. The triangle is a marginal case. The radii of the circles in the successful cases are scaled by E_{exp} .

the inflow velocity of the accreting matter, on the other hand, the Alfvén waves propagate outwards at the speed v_A . This means that the inflowing matter is heated up by the dissipation of Alfvén waves just when it reaches the vicinity of the stalled shock wave.

The luminosity, L_A , of Alfvén waves at the PNS surface can be estimated under the assumption of spherical symmetry as follows:

$$L_A = \frac{1}{2} \rho_0 \delta v_{\perp,0}^2 v_{A,0} 4\pi r_0^2$$

$$\approx 10^{52} \text{erg s}^{-1} \left(\frac{\rho_0}{10^{11} \text{g cm}^{-3}} \right)^{1/2} \left(\frac{c_{s,0}}{0.1c} \right)^2 \left(\frac{\epsilon}{0.2} \right)^2$$

$$\left(\frac{B_{r,0}}{2 \times 10^{15} \text{G}} \right) \left(\frac{r_0}{50 \text{km}} \right)^2. \quad (10)$$

This implies that the emission of Alfvén waves for ~ 100 ms gives the energy injection of $\sim 10^{51}$ erg. In most cases, almost all the energy of Alfvén waves is absorbed thanks to the trapping of Alfvén waves just mentioned. This is also confirmed by the comparison with the simulation results. We can calculate the total heating rate (erg g^{-1}) by multiplying the heating rate per unit mass ($\text{erg g}^{-1} \text{s}^{-1}$; the top panel of Figure 3) by the density ($\approx 10^9 \text{g cm}^{-3}$) and volume of the heating region, $4\pi r^2 \Delta r$, where $\Delta r (\approx 100 \text{km}$ in Figure 3) is the thickness of the heating region. We thus obtain the total heating rate of $\approx 10^{52} \text{erg s}^{-1}$ for model V, which indicates that the injected Alfvén wave luminosity is mostly used for heating up the stalled shock.

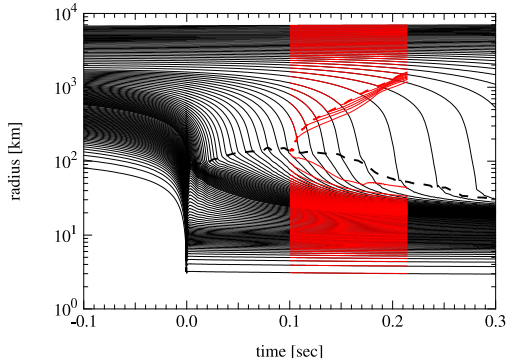


FIG. 2.— Time evolutions of mass shells. Model V with Alfvén wave heating ($B_{r,0} = 2 \times 10^{15} \text{G}$ & $\epsilon = 0.2$; red lines) is superimposed on the original non-magnetized model (black lines). The time is measured from the core bounce. The dashed lines show the locations of shock front.

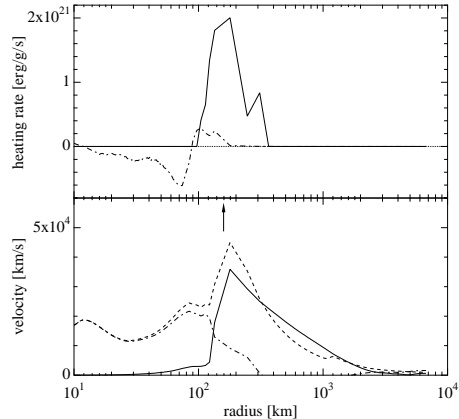


FIG. 3.— *Top*: The heating (cooling for negative values) rates from neutrino (dot-dashed) and Alfvén waves (solid) of model V at $t = 100 \text{ms}$. The arrow indicates the location of the shock front. *Bottom*: The distributions of $-v_r (> 0)$ (solid), v_A (dashed), and $v_r + v_A$ (dot-dashed) at $t = 100 \text{ms}$ for model V.

The regions with fast accretion velocities are preferentially heated up and eventually start to move outwards, provided L_A is sufficiently large. Once the stagnated shock wave is re-launched, the heating is reduced because Alfvén waves become untrapped again. For larger L_A the shock revival occurs earlier and the duration of heating, $\Delta \tau_A$, is shorter. As a result, E_{exp} (roughly $\propto L_A \Delta \tau_A$) is not very sensitive to $B_{r,0}$ and ϵ . In fact, although L_A of model IX is larger than that of model V by more than a factor of 3, the difference in E_{exp} is less than a factor of 2. In this sense the Alfvén wave mechanism is self-regulating. Interestingly, the acoustic wave mechanism is also claimed to be self-regulating (Burrows et al. 2006) though the regulating mechanism is different: the generation of the acoustic waves continues until the shock is revived and matter ceases to accrete.

Model VII is exceptional among the explosion cases, giving a very small explosion energy. The Alfvén wave heating operates in a much outer region in this case because Alfvén waves become nonlinear, $\delta B_{\perp}/B_r > \alpha$, only after crossing the shock wave owing to the large $B_{r,0}$ and small ϵ . A sizable fraction of L_A leaks out of the stalled shock wave and a tiny amount is ejected with a quite small E_{exp} .

The models with $B_{r,0} = 1 \times 10^{15} \text{G}$ produce no explosion. This is first because L_A itself is small (mainly models I & II) owing to the small $B_{r,0}$ and second because the dissipation of Alfvén waves occurs too early (mainly model III). Note in particular that L_A of model III is larger than those of the explosion cases V and VII. In this case Alfvén waves dissipate in the inner region and the temperature is increased there. As a result, the energy deposited by Alfvén waves is mostly converted to neutrino emission in this case.

4. DISCUSSION

In this section, we discuss more in detail the validity of the assumptions and approximations employed for the background magnetic field and the formulation of Alfvén wave propagations in this paper.

4.1. Magnetic Field

Since we have seen that strong magnetic fields ($B_{r,0} \gtrsim 2 \times 10^{15} \text{G}$), which are of the magnetar scale and much

larger than those for the ordinary radio pulsars, are necessary to revive the stalled prompt shock by the Alfvén wave heating, one may think that the mechanism considered in this paper is only applicable to magnetar-forming supernovae and the Alfvén wave is not a major ingredient in the ordinary SN explosion. We cautiously note, however, that this strong magnetic field is required not for the ordinary neutron stars as we observe them but for the PNSs in their very infancy. As a matter of fact, there is a speculation that the magnetic fields of nascent PNS may be temporarily very strong and then decrease to the ‘normal’ value ($\sim 10^{12}$ G) by energy releases occurring during the SN explosion and later evolution (Wheeler, Meier, & Wilson 2002). If this is true, the Alfvén wave mechanism may work in a larger population of core-collapse SNe.

The origin of such strong magnetic fields is still controversial. One possibility is referred to as the fossil origin hypothesis: the strong magnetic field in compact stars is simply a consequence of the compression of the magnetic field that already exists in OB progenitors prior to the gravitational collapse. In fact, several magnetic massive stars have been observed to have a magnetic field whose average dipole-field strength is ~ 1000 G (Neiner et al. 2003; Hubrig et al. 2006; Donati et al. 2006). The total magnetic flux of these stars is comparable to that of a typical magnetar (Ferrario & Wickramasinghe 2006), which implies that an additional generation and/or amplification of magnetic fields will not be necessary to obtain a highly magnetized compact remnant for these stars.

Another possibility is an amplification of weak magnetic fields by the MRI (Akiyama et al. 2003; Masada, Sano, & Takabe 2006), in which stellar rotation plays a key role, winding poloidal fields and driving the instability. In this scenario, toroidal magnetic fields are efficiently produced, whereas in the case of the fossil origin, we expect the radial component of magnetic field is dominant.

Since the Alfvén wave carries energy along a field line, we are interested only in open magnetic flux tubes that extend beyond the stalled shock wave. As the simplest configuration, we have considered radial magnetic fields and neglected the toroidal component in this paper. As mentioned above, the approximation is justified if the magnetic field is of fossil origin and the progenitor is a slow rotator. Even if the progenitor core is a rapid rotator, our models will be still applicable to the polar region, where the effect of rotation is not strong and the toroidal magnetic fields are less important, although the radial dependence of the field strength may need elaboration.

In the equatorial region of a rapidly rotating supernova core, on the other hand, the situation is much more complicated. Field lines are not directed radially in general and some of them may be closed, as expected for the dipole configuration. In addition, the continuous downward advection of magnetic fields may cause reconnections in the PNS, which in turn will open up some field lines again. In any case the toroidal component will be dominant over the radial component (Burrows et al. 2007) and we need to include the effects of these spiral magnetic fields as well as rotation itself in discussing the Alfvén wave heating in supernova cores quantitatively,

which will be the future task.

4.2. Alfvén Wave Propagation

The treatment of Alfvén waves in this paper is admittedly a crude approximation. We employ the non-relativistic, steady-state, and WKB approximations for describing the propagation of Alfvén waves. Among the assumptions, the non-relativity is adequate for the Alfvén waves launched from the PNS surface since the relativistic corrections are indeed minor outside the PNS. The steady-state approximation is also reasonable because the Alfvén transit time is shorter than the expansion time of accreting matter; while the expansion time-scale of the ejecta is $\sim 50 - 100$ ms, the time for the Alfvén wave to travel from the PNS surface ($r \approx 50$ km) to the wave trapping region around the stalled shock ($r \approx 200$ km) is ~ 10 ms for $v_A \approx 2 \times 10^4$ km s $^{-1}$.

The WKB approximation is acceptable if the wavelength is shorter than the scale height of the background. The typical period of Alfvén waves generated in the PNS is supposed to be $\tau \sim 1$ ms, corresponding to the dynamical time-scale. Then, the wavelength becomes, $\lambda \approx v_A \tau \sim 2 \times 10^4$ km s $^{-1} \times 10^{-3}$ s ~ 20 km. Since the density scale height is shortest near the PNS surface and is $H_\rho \approx 30$ km, Alfvén waves might be partially reflected there due to the deformation of the wave shape. In more detailed study, this effect should be taken into account together with the wave pressure ignored in this paper.

5. CONCLUSION

In this paper we have studied the matter heating by Alfvén waves in the post-bounce supernova core and its implications for the shock revival. In order to elucidate the essential features of the mechanism quantitatively, we have done a couple of 1D dynamical simulations, neglecting rotation and toroidal magnetic fields but employing the Alfvén-wave heating rate based on our model of the nonlinear damping of Alfvén waves.

We have found that if the surface magnetic field strength is $\gtrsim 2 \times 10^{15}$ G and if the surface velocity fluctuation is $\epsilon \gtrsim 0.2$, which corresponds to $\delta v_{\perp,0} \gtrsim 6 \times 10^3$ km s $^{-1}$, the stalled shock is revived by the Alfvén wave heating with a canonical explosion energy, $E_{\text{exp}} \gtrsim 10^{51}$ erg. The current mechanism is self-regulating in the sense that the explosion energy is not very sensitive to the surface field strength and initial velocity fluctuation as long as they satisfy the above conditions. The above strong magnetic field is not a requirement for the ordinary neutron stars as observed but for the PNSs in their infancy. If magnetic fields decay through their subsequent evolution (Wheeler et al. 2002), the Alfvén waves may play an important role in a larger population of core-collapse SNe.

It has been also found that the wave trapping is essential in localizing the Alfvén wave heating in the vicinity of the stalled shock wave as well as in regulating the explosion energy. In fact, if the magnetic field is weaker ($\lesssim 2 \times 10^{15}$ G), the Alfvén wave heating takes place much closer to the PNS because the Alfvén wave becomes nonlinear earlier on at smaller radii. Then no shock revival occurs because the dissipated energy is mostly lost by neutrino cooling; neutrino emissions are enhanced in this case. If the initial velocity fluctuation is smaller ($\epsilon \lesssim 0.2$) with a stronger magnetic field, $\gtrsim 3 \times 10^{15}$ G, on

the other hand, most of Alfvén waves propagate through the stalled shock region with only a small amount of energy being deposited, which then results in a weak explosion with $E_{\text{exp}} < 10^{51}$ erg. It is noted, however, that even in these cases the Alfvén wave heating will be still important in supplementing the neutrino heating.

As a first step, the models presented in this paper have much room for sophistication. Among other things, as mentioned repeatedly, rotation and toroidal magnetic fields should be somehow taken into account. In reality, the Alfvén wave mechanism probably works in cooperation with these processes. This will require multi-dimensional numerical modellings and will be the future work.

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